# Multi-conjugate adaptive optics for a new generation of giant telescopes

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# Multi-conjugate adaptive optics for a new generation of giant telescopes

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## ABSTRACT

A handful of groups around the world are actively working on the development of the next generation of telescopes of 30 m diameter and more. Present implementations of adaptive optics will be inadequate to realize the full resolving power of these new instruments in imaging and spectroscopy. Instead, multi-conjugate adaptive optics (MCAO) systems are being contemplated. We explore here the application of MCAO using laser guide beacons, to a 30 m telescope. Using a new simulation code, we show that reliance on the expensive lasers needed to generate sodium resonance beacons can be reduced through the use of refocused Rayleigh laser guide stars at much lower cost.

We show that in the geometric optics approximation, modes of the phase distortion of order less than or equal to the number of deformable mirrors in the MCAO are correctable with no isoplanatic error. A new figure of merit is derived which predicts the relative ability of a chosen beacon/deformable mirror architecture to sense and correct wavefront aberration, based solely on knowledge of the optical geometry and the statistics of the aberration to be corrected. Numerical simulation can therefore be minimized by avoiding the exploration of unpromising beacon arrangements.

**Keywords:** Multi-conjugate adaptive optics, giant telescopes, laser guide stars, tomography

## 1. INTRODUCTION

All current and planned telescopes in the 6 to 10 m range now include or are planning adaptive optics. The potential gains in sensitivity and resolution will be even higher for the next generation of telescopes of 20 m diameter and more. But just as our telescopes take the next big step forward, so too must the adaptive optics. A single sodium laser guide star (SLGS), for example, is not adequate to correct even a narrow field of view at such large aperture size because of focus anisoplanatism. The goal will be to provide imaging at the diffraction limit of resolution, now sharper than ever, over a wider field, and at shorter wavelengths than our current tools can manage. This must all be done while maintaining a point-spread function that is as stable as possible in both field of view and time.

To achieve this goal, multi-conjugate adaptive optics (MCAO), originally proposed by Beckers[1] will be required. In this paper we explore the performance of a 30 m telescope that takes advantage of refocused Rayleigh laser guide stars (RLGS).[2,3] Beacons at the low altitudes ($< 25$ km) typical for RLGS have not been attractive for large telescopes in the past because of the large errors introduced in the measurement of the wavefront by focus anisoplanatism. Furthermore, the depth of focus of a 30 m telescope observing a target at 25 km is no more than $\sim 100$ m, so the wavefront sensor (WFS) must be restricted to an integration time of $< 1\ \mu$s to avoid image blur. To achieve sufficient signal-to-noise ratio on the sensor would then require lasers of enormous power and cost.

The first difficulty is largely overcome when multiple beacons are used to sense the wavefront, so that the volume of turbulent air through which natural starlight travels is also fully sampled by laser light. Since tomographic wavefront sensing for MCAO demands multiple beacons anyway, this condition is naturally satisfied.

The question of range gating can also be addressed by actively tracking the focal plane of the return from a moving pulse of beacon light. By keeping the image of the beacon sharply focused, the WFS integration time may be extended by two orders of magnitude.[2] The laser power requirement then becomes no more than a few watts for wavefront compensation in the near IR which is entirely practical.

To explore the performance to be expected from a telescope using refocused RLGS beacons, a new simulation code has been written, described elsewhere in these proceedings.[4] Results are presented here for a number of representative beacon geometries. To begin though, we present some theoretical work which describes requirements that must be satisfied by the sensing/correcting geometry of the MCAO system, and a new way to make a qualitative assessment of candidate geometries rapidly without resorting to time-consuming numerical modeling.

## 2. ISOPLANATIC CORRECTION WITH MCAO

In general, when wavefront phase errors in the light coming from a point source are corrected, phase errors in the light from any nearby point are not perfectly corrected, because the light has followed a different path through the atmosphere. MCAO addresses the problem by reproducing in the telescope optics the three-dimensional structure of the atmospheric phase aberration, thereby allowing cancellation of phase errors in any direction within some given field of view. In fact, as we show here, a system with some number $M$ of deformable mirrors can reproduce atmospheric aberration for all modes of polynomial degree $\leq M$ with no residual isoplanatic error.

In the following analysis, we assume the geometric optics approximation in which phase errors are accumulated along a ray path without regard to refraction. This assumption is reasonable for the weak turbulence typically encountered in near-vertical observations at a good astronomical site.

### 2.1. Choice of basis set

We assume that the phase is a well-behaved function, i.e. continuous, single-valued, and with no discontinuities. This restricts us to the domain of weak turbulence: in strong turbulence it is quite possible for phase wraps to occur.[5]

Most work on the analysis of atmospheric phase perturbations relies on the Zernike basis set to describe the aberrated phase front $\phi(\rho, \theta)$. A complete description is given by coefficients $\zeta_{nm}$ of the Zernike polynomials $Z_{nm}$ where the radial and azimuthal orders of the polynomial are respectively $n$ and $n - 2m$;

$$\phi(\rho, \theta) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \zeta_{nm} Z_{nm}(\rho, \theta) \quad . \tag{1}$$

Zernike's functions have the very convenient property of orthonormality over the unit circle, and the low-order functions are easily related to classical aberrations that opticians learn to recognize. They are less useful in the present context however, where we must consider aberrations not within the unit circle, but over a smaller circular window at an arbitrary location within the unit circle. This geometry arises from the footprint of beacon light approaching the telescope on a metapupil whose diameter is determined at a given height by the diameter of the telescope and the field of view.

We choose instead a simple polynomial representation $\phi(x, y)$ in rectangular coordinates as the most convenient for the present analysis. Recognizing that $Z_{nm}$ can be written as a polynomial of degree $n$ in $x$ and $y$, we can approximate any function $\phi$ through order $N$ by

$$\phi \approx \sum_{n=0}^{N} \sum_{m=0}^{n} \zeta_{nm} Z_{nm} \equiv \sum_{i=0}^{N} \sum_{j=0}^{N-i} \alpha_{ij} x^i y^j \tag{2}$$

where the coordinates $x$ and $y$ are expressed in units of the telescope radius $R$.

### 2.2. Wave front compensation

The atmospheric phase aberration in the telescope pupil accumulated by light from a star at angular position $(\xi, \upsilon)$ in the field may be written as a general polynomial in four variables:

$$\Phi_*(x, y; \xi, \upsilon) = \int_0^\infty \sum_{i=0}^\infty \sum_{j=0}^\infty \alpha_{ij}(h) X^i Y^j dh \tag{3}$$

where $h$ is the height in the atmosphere, and $\alpha_{ij}(h)$ are the atmospheric coefficients of the phase aberration introduced per unit height. It will be convenient to express quantities of height normalised by $R$, in which case we shall write for instance $\hat{h} = h/R$. We set $X = x + \xi\hat{h}$ and $Y = y + \upsilon\hat{h}$ as the coordinates at height $h$ of a ray coming from $(\xi, \upsilon)$ and intersecting the pupil at point $(x, y)$. Note that eq. 3 contains a piston term $i = j = 0$ which for notational convenience we leave in for the time being, and also that we have not made any assumption about the shape of the telescope aperture.

In general, our MCAO system will have only a finite number of degrees of freedom to correct the wave front. We then write $\Phi_*^N$ as the component of the phase aberration containing modes from orders 1 through $N$ that we can hope to correct:

$$\Phi_*^N(x, y; \xi, \upsilon) = \int_0^\infty \sum_{i=0}^N \sum_{j=0}^{N-i} \alpha_{ij}(h) X^i Y^j dh \tag{4}$$

Expanding out the $X^i$ and $Y^j$ we find

$$\Phi_*^N(x, y; \xi, \upsilon) = \sum_{i=0}^N \sum_{k=0}^i \binom{i}{k} x^{i-k} \xi^k \sum_{j=0}^{N-i} \sum_{l=0}^j \binom{j}{l} y^{j-l} \upsilon^l \int_0^\infty \alpha_{ij} \hat{h}^{k+l} dh \tag{5}$$

$$= \sum_{k=0}^N \sum_{l=0}^{N-k} x^k y^l \sum_{i=k}^{N-l} \binom{i}{k} \xi^{i-k} \sum_{j=l}^{N-i} \binom{j}{l} \upsilon^{j-l} \int_0^\infty \alpha_{ij} \hat{h}^{i+j-k-l} dh \tag{6}$$

where $\binom{i}{k}$ is the binomial coefficient $i!/k!(i-k)!$. The expansion demonstrates that an atmospheric mode of given degree arising at altitude $\hat{h} \neq 0$ will produce aberrations in off-axis starlight in many modes of equal and lower degree. A well-known example is the "breathing" mode, or radial motion of off-axis stellar images in compensated imaging from single-conjugate AO systems. This form of anisoplanatism is caused by uncorrected field-dependent tilt terms arising from high-altitude aberration.[6]

The phase applied by a given deformable mirror in the beam train is a function of two variables only, namely $x$ and $y$. If we apply a general phase function of order $N_m$ to mirror $m$ we can write the function as

$$\psi_m(x, y) = \sum_{i=0}^{N_m} \sum_{j=0}^{N_m-i} \beta_{ij,m} x^i y^j \quad , \tag{7}$$

where the coefficients $\beta_{ij,m}$ are to be chosen to cancel the atmospheric aberration. Let a set of mirrors $m = 1...M$ be conjugated to heights $H_m$ in the optical beam train. The correction phase $\Psi$ applied by these mirrors in direction $(\xi, \upsilon)$ is

$$\Psi(x, y; \xi, \upsilon) = \sum_{m=1}^M \psi_m(x + \xi\hat{H}_m, y + \upsilon\hat{H}_m) \tag{8}$$

$$= \sum_{m=1}^M \sum_{i=0}^{N_m} \sum_{j=0}^{N_m-i} \beta_{ij,m} (x + \xi\hat{H}_m)^i (y + \upsilon\hat{H}_m)^j \quad . \tag{9}$$

If we set $N_m = N$ for all $m$, that is, all the mirrors correct to the same degree, then

$$\Psi(x,y;\xi,\upsilon) = \sum_{k=0}^{N}\sum_{l=0}^{N-k} x^k y^l \sum_{i=k}^{N-l}\binom{i}{k}\xi^{i-k}\sum_{j=l}^{N-i}\binom{j}{l}\upsilon^{j-l}\sum_{m=1}^{M}\beta_{ij,m}\hat{H}_m^{i+j-k-l} \quad . \tag{10}$$

Now we assume that $\beta_{ij,m}$ can be found such that $\Psi = \Phi_*^N$. Equating coefficients of like terms $x^k y^l$ in eqs. 6 and 10 we can say

$$\sum_{i=k}^{N-l}\binom{i}{k}\xi^{i-k}\sum_{j=l}^{N-i}\binom{j}{l}\upsilon^{j-l}\sum_{m=1}^{M}\beta_{ij,m}\hat{H}_m^{i+j-k-l} = \sum_{i=k}^{N-l}\binom{i}{k}\xi^{i-k}\sum_{j=l}^{N-i}\binom{j}{l}\upsilon^{j-l}\int_0^\infty \alpha_{ij}\hat{h}^{i+j-k-l}dh \quad . \tag{11}$$

Eq. 11 shows explicitly the field dependence of the contributions to the coefficient of $x^k y^l$ in the sensed aberration which result from atmospheric aberration terms $x^i y^j$ of equal and higher order. We see also that the desired mirror coefficients $\beta_{ij,m}$ are to be found in terms of moments of the atmospheric aberration coefficients $\alpha_{ij}(h)$.

If we insist on strict isoplanicity, then eq. 11 must hold for any values of $\xi$ and $\upsilon$ and we can say further

$$\sum_{m=1}^{M}\beta_{ij,m}\hat{H}_m^n = \int_0^\infty \alpha_{ij}\hat{h}^n dh \qquad \text{where } \; 0 \le n \le i+j \quad . \tag{12}$$

For a given atmospheric aberration $x^i y^j$, eq. 12 gives the mirror coefficients required to correct all the terms of degree $n$ seen by off-axis starlight. In particular, for the highest order atmospheric aberrations where $i + j = N$, eq. 12 represents a set of $N + 1$ simultaneous equations. A complete solution, justifying the assumption leading to eq. 11, then requires exactly $N + 1$ variables $\beta_{ij,m}$, or $M = N + 1$.

However, the constraint imposed by eq. 12 with $n = i + j$ implies $k = l = 0$ from eq. 11. This term in eqs. 6 and 10 merely represents piston and is of no account in terms of the imaging system's performance. Ignoring this term also in eq. 12 reduces the number of independent atmospheric integrals from $N + 1$ to $N$. It is therefore necessary only that $M = N$ for fully isoplanatic correction of $N^{\text{th}}$ order atmospheric aberration. An inductive argument then shows that $N$ mirrors are also sufficient to correct all lower orders simultaneously.

We have made no assumptions in this analysis about the vertical distribution of turbulence in the integrals in eq. 12. We conclude then that for any $C_n^2$ profile, a system of $N$ DMs can control modes of phase aberration through $N^{th}$ order at all field angles. In a strict mathematical sense, this is true for any set of DM conjugates $H_m$ provided only that they are all distinct, that is, no two DMs are conjugated to the same height. In practice, optimum conjugates for any set of DMs will be determined by considering isoplanatic errors introduced by modes of order $> N$.

Since by hypothesis we can apply all modes up to order $N$ to each DM, we in fact have an excess of degrees of freedom for controlling modes of order $< N$. This means the tomographic reconstructor matrix which maps beacon measurements onto the DMs must reject the excess modes to maintain a stable system. How many of these modes are there? Exactly $n$ DMs are needed for fully isoplanatic control of modes of order $n$, of which there are $n+1$ per DM. If we have $N$ DMs, then we must reject $N(n+1) - n(n+1) = (N-n)(n+1)$ modes. The total number to be removed, including $N$ piston modes, is then

$$\sum_{n=0}^{N}(N-n)(n+1) = \frac{1}{6}N(N+1)(N+2) \quad . \tag{13}$$

## 3. CORRECTING ABILITY OF A TOMOGRAPHIC RECONSTRUCTOR

The tomographic reconstructor is a matrix that relates the individual wavefronts derived from the wavefront sensors to the signals needed to drive the MCAO system's DMs. Individual wavefronts will be recovered in the usual way for single conjugate AO systems, using a reconstructor matrix $\boldsymbol{R}$ applied to a measured vector of parameters $s_b$ from a wavefront sensor looking at beacon $b$:

$$a_b = \boldsymbol{R} s_b \quad , \tag{14}$$

where $a_b$ is a vector of coefficients of the modes of the chosen basis set. All the recovered wave front coefficients are assembled into a single vector,

$$A = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \end{bmatrix} \quad . \tag{15}$$

A tomographic reconstructor matrix $\boldsymbol{T}$ is then applied to obtain a vector of coefficients to be applied to the DMs:

$$B = \begin{bmatrix} \beta_{ij,0} \\ \beta_{ij,1} \\ \vdots \end{bmatrix} = \boldsymbol{T} A \quad . \tag{16}$$

The matrix $\boldsymbol{T}$ can in principle be obtained by inverting a matrix $\boldsymbol{F}$ of measured influences of the DMs on the WFSs, in a manner very similar to that in which $\boldsymbol{R}$ is normally obtained. Degenerate modes, enumerated in eq. 13, must be projected out of $\boldsymbol{T}$, as well as modes which are poorly sensed by the chosen arrangement of beacons.

A well-designed MCAO system places DMs and beacons such that atmospheric aberration can be well corrected. What we desire is that all modes of the aberration that introduce significant power be well sensed by the beacons. We can compute a figure of merit for a chosen beacon/DM geometry, using just the corresponding tomograph itself and knowledge of the atmospheric statistics, which relates directly to the tomograph's ability to reduce phase aberration. We start by conjugating our DMs to the mean heights of selected slabs of the atmosphere. Then we measure or compute the influence $\boldsymbol{F}$ on the WFSs of the mirror modes applied to each of the DMs, and decompose $\boldsymbol{F}$ with singular value decomposition to find the matrices $\boldsymbol{W}$, $\boldsymbol{\Lambda}$, and $\boldsymbol{M}$ such that

$$\boldsymbol{F} = \boldsymbol{W} \boldsymbol{\Lambda} \boldsymbol{M}^T \tag{17}$$

where $^T$ represents the transpose operator.

The matrix $\boldsymbol{\Lambda}$ is diagonal with the elements being the singular values of $\boldsymbol{F}$ which indicate how well the atmospheric modes defined by the columns of $\boldsymbol{M}$ are sensed. That is, if the mirrors are commanded to conform to a set of modes corresponding to column $n$ of $\boldsymbol{M}$, $M_n$, then the WFSs will record a response defined by $W_n$ with magnitude $\Lambda_n$. The singular values therefore quantify how well each mode represented by $\boldsymbol{M}$ is measured.

We then seek the power in the wavefront aberration associated with each basis vector of $\boldsymbol{M}$, averaged over time and over the MCAO system's field of view. We compute

$$\boldsymbol{P}_{ATM} = \boldsymbol{M}^T \boldsymbol{C}_B \boldsymbol{M} \quad . \tag{18}$$

$\boldsymbol{C}_B$ is the covariance matrix of the mirror commands $B$. For the results presented below, Zernike modes were used as the basis set, although this would be better done with the statistically-independent Karhunen-Loeve functions. The diagonal elements of $\boldsymbol{P}_{ATM}$ represent the variance of the applied phase, that is, the atmospheric power, in the corresponding column vector of $\boldsymbol{M}$. Implicit in eq. 18 is an average over the field of view, since each column of $\boldsymbol{M}$ comprises a concatenation of vectors of modal coefficients to be applied to the various DMs, over the full size of the metapupil.

Finally, the figure of merit $g$ is computed as the correlation between the atmospheric power represented by $\boldsymbol{P}_{ATM}$ and the sensitivity of the tomograph represented by $\boldsymbol{\Lambda}$,

$$g = \frac{diag(\boldsymbol{P}_{ATM}) \cdot diag(\boldsymbol{\Lambda})}{diag(\boldsymbol{\Lambda}) \cdot diag(\boldsymbol{\Lambda})} \quad . \tag{19}$$

As an illustration of the utility of $g$, we have modeled the case of a 10 m telescope corrected with 3 DMs, each driven in 230 modes. The simulation code is described elsewhere.[4] Wavefront sensing was done with 5 SLGS and 5 RLGS arranged as pentagons on the same radius, but clocked by 36°. A single NGS was modeled for tilt measurement. The rms residual wavefront error after compensation was calculated over a field of view of 40 arcsec radius, using 100 random atmospheric realizations. The residual wavefront error and $g$ were both calculated as a function of the angular radius of the beacon constellation, as shown in figure 1. In this simulation, the maximum of $g$ accurately predicts the best performance given by the minimum of the residual wavefront error. We are currently exploring the use of $g$ as a predictor for other system parameters.

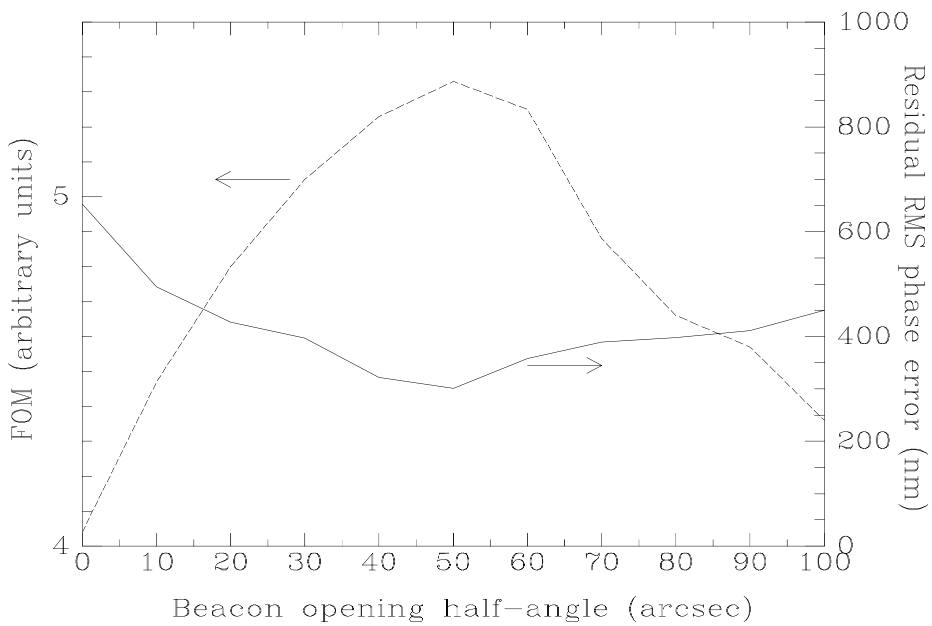


Fig. 1. Demonstration of the performance of the figure of merit calculated from eq. 19. In this instance, a system of 5 sodium and 5 Rayleigh beacons was used to compensate a 10 m telescope with 3 DMs. The FOM accurately predicts the beacon opening angle which minimizes the rms phase aberration averaged over a field of 40 arcsec.

## 4. NUMERICAL RESULTS

Numerical simulations have been carried out to explore the performance of a range of MCAO beacon configurations for 30 m class telescopes using the new modeling code. Atmospheric turbulence and DM corrections are represented as vectors of Zernike coefficients. The model analytically computes the influence on the WFSs of each mode applied to both the DMs and the layers of turbulence in the atmosphere. The tomographic reconstructor matrix is then computed by taking the product of the SVD inverse of the DM influence matrix and the atmospheric layer influence matrix.

Random turbulence Zernike coefficient vectors were generated for a model atmosphere based on scidar measurements[7] recorded above Mt. Hopkins, and using Kolmogorov statistics for Zernike order $1 \leq N \leq 30$. Read noise with Gaussian statistics and Poisson photon noise are simulated for a Hartmann-Shack sensor and added to the noise-free wavefront corrections.

Figure 2 illustrates a sample beacon configuration consisting of an outer ring of five sodium beacons at 95 km arranged in a regular pentagon at a radius of 1.3 arcmin from the optical axis. A second inner ring of five Rayleigh beacons at 35 km at a radius of 0.5 arcmin is clocked at an angle of 36° with respect to the sodium ring.

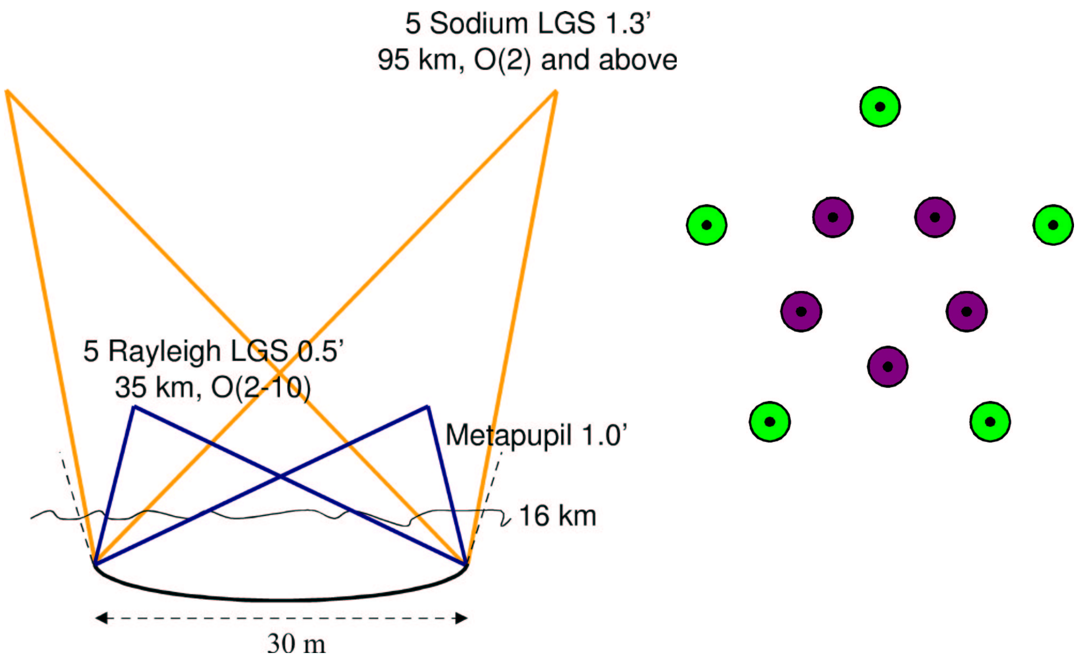


Fig. 2. Representative schematic beacon geometry studied in the numerical simulations. In this example, an inner ring of 5 RLGS at 0.5 arcmin is surrounded by a second ring of SLGS at 1.3 arcmin.

A tri-conjugate MCAO system was simulated for a 30 m telescope which was assumed to have a 10% central obstruction and a FOV of 1 arcmin radius. Turbulence for a 7-layer model atmosphere for Mt. Hopkins was simulated using 495 Zernike modes, corresponding to order 30. The 30 m MCAO system included an adaptive secondary which formed the pupil (0 km), a second DM conjugated at 3.4 km, and a third DM conjugated at 15.9 km. The adaptive secondary corrected 495 Zernike modes, or radial orders 1 (tip/tilt) through 30. The second and third DMs corrected 493 modes (orders 2 through 30) and 490 modes (orders 3 through 30) respectively.

A range of beacon configurations were simulated to evaluate the expected performance of a MCAO system using sodium beacons exclusively as well as a combination of sodium and Rayleigh beacons. Each SLGS was modeled as the return from a 10 mJ pulse at a mean height of 95 km. The Rayleigh beacons were modeled as the integrated photon return from 35–45 km from a 5 mJ pulse at 351 nm. Signals were sensed from the SLGS by 26 × 26 Hartmann-Shack WFS reconstructing all Zernike modes in orders 2–30. Additional 10 × 10 Hartmann-Shack WFS sensed Zernike modes in orders 2–10 from the RLGS. Each WFS was assumed to have 50% detected quantum efficiency and a read noise of 3 $e^-$ RMS. Finally, an additional quad sensor recorded tilt modes from a natural guide star at $R$ magnitude 17.

Three geometries have been examined, and results are plotted in figure 3 expressed as RMS residual wavefront error as a function of field angle. First, a pentagon of SLGS on a radius of 1.3 arcmin was used. Although the compensated field of view is 45 arcsec, the level of wavefront correction is not very encouraging. With the addition of five more SLGS closer to the axis at 0.8 arcmin, the degree and uniformity of correction are both greatly improved. Finally, the inner ring of SLGS was replaced by a constellation of eight RLGS. Collectively the RLGS sensed a factor of 5 fewer modes than the SGLS they replace, and yet the level of performance was maintained at almost the same level.

We have not attempted here to find the optimal arrangement of the hybridized LGS constellation. That is reserved for future work. Nevertheless, this result demonstrates that RLGS can be expected to play a crucial role in MCAO for giant telescopes by reducing the number of sodium lasers required.

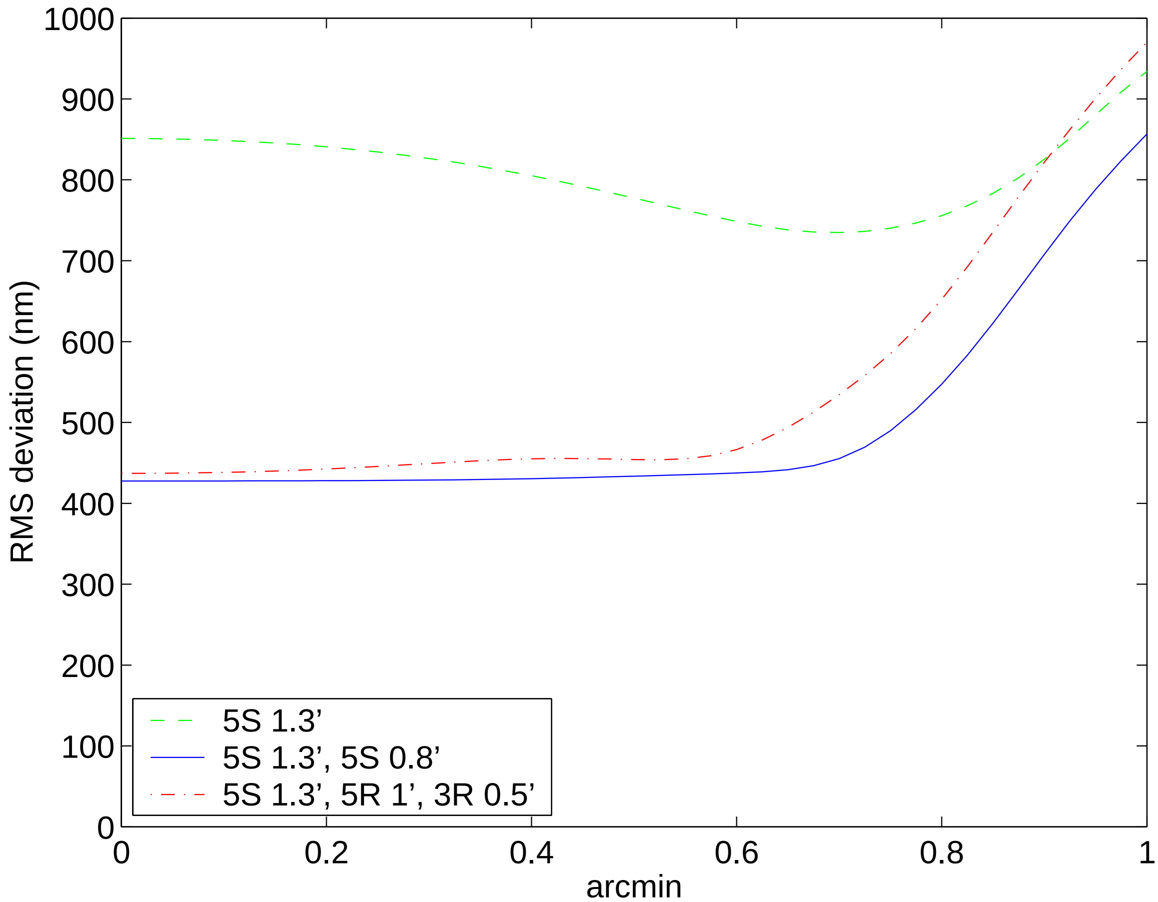


Fig. 3. Sodium only and sodium plus Rayleigh performance vs. radius of the corrected field of view for the simulated 30 m MCAO system.

## 5. CONCLUSIONS

The theoretical work presented here shows that with appropriate wavefront sensing, anisoplanatism in the lowest order, highest power modes of the atmospheric aberration can be completely eliminated. We have also begun to explore the potential for tomographic matrices to be compared without the need for extensive numerical simulation through the computation of a figure of merit based on the tomograph's inverse and knowledge of the aberration statistics.

When LGS are used as beacons for MCAO, it is highly desirable to use layers of beacons at two different heights. The height diversity may be provided by multiple NGS and a single layer of LGS, or as explored here, LGS at two altitudes. In this way, the vertical distribution of low-order modes, which would otherwise be formally insoluble, becomes possible, and performance can be improved. Indeed modes of order not higher than the number of DMs can in principle be compensated at all field angles.

Height diversity of LGS can be achieved in a number of ways: multiple exposures of the same pulse of beacon light from different heights, either with dynamic refocus,[2] or as input to a phase diverse algorithm[8] are two possibilities. The solution explored in this paper is the use of both RLGS and SLGS. We see that despite the comparatively low altitude of the RLGS, and the inherently large focus anisoplanatism of a single beacon, when used in such a combination they are very powerful: the requirement on the number of sodium lasers can be significantly reduced. At this stage in the development of 30 m telescopes, when it appears likely that reliable sodium lasers will cost well over $1M each, the option of replacing some of them with lasers costing 1/10 as much is an attractive alternative.


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