Dynamic Refocus for Rayleigh Beacons

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ABSTRACT

Rayleigh beacons for adaptive optics at the LBT are being considered at Arcetri and Arizona. Their real strength comes when multiple lasers are to be used for atmospheric tomography, as discussed by Angel and Lloyd Hart[1] and Brusa et al.[2] Strong photon fluxes from high altitude are potentially available, because if a short, diffraction limited laser pulse is projected as a narrow beam, the scattering at any instant appears no larger than the seeing disc over a large range of height. But a way to capture the wavefront from a fast-moving source must be found. Here we show that a large gain in signal strength can be achieved by dynamic refocus of the telescope, simply by axial motion of the detector at uniform speed. For example, for an 8.4 m telescope with focus fixed at 20 km altitude, a range gate of 0.5 km would be required to limit blurring to 1 arcsec. But with dynamic refocus the gate can be broadened to accept virtually all the scattering from above 20 km with no increase in image blur, for an order of magnitude gain in signal strength. Comparing the yield of lasers of the same power, a 351 nm Rayleigh beacon with dynamic refocus produces the same strength photon flux from a mean height of 27 km as a perfectly optimized sodium beacon (when the sodium layer has median column density). For atmospheric tomography, about twice as many Rayleigh as sodium beacons will be needed. This would seem to be a small price to pay for the convenience of using lasers that are commercially available, relatively simple, reliable and much less expensive.

Keywords: adaptive optics, laser beacons, Rayleigh scattering

1. INTRODUCTION

Virtually all planning for laser guide stars for astronomy is based on resonance scattering by the sodium layer at 95 km elevation. The advantages are clear: the great height of the layer minimizes the error in wavefront from focus anisoplanatism (the cone effect) while the fraction of energy scattered by the layer is relatively high, 3% for the average sodium column density. For comparison, 1% of an ultraviolet (0.350 nm) beam travelling straight up is scattered above 25 km height.

However it is proving difficult to achieve in practice the precisely tuned, high power, reliable lasers at 589 nm wavelength needed for a sodium resonance guide star. Saturation reduces the scattered fraction unless the laser is cw or has a high duty cycle. Furthermore, the return strength is unpredictable, because of fluctuations by up to an order of magnitude in sodium column density. For some purposes the reliable return and low relative cost of Rayleigh beacons may be preferred. In particular, if multi-layer analysis of atmospheric turbulence is to be undertaken with multiple beacons, then the cone effect can be allowed for in the tomographic solution and the lower altitude is not a serious drawback.

Rayleigh beacons with adaptive optics have yielded near diffraction limited near-red images from a 1.5 m telescope.[3] However, in the past performance has been limited by poor beam quality in the lasers and defocused or weak signals set by depth of field limitations in both the projected and returned beams. New lasers with near diffraction limited quality remove the depth of field restriction for the projected beam, as we show below. For the return beam, there are a number of possible ways to increase the depth of field to improve beacon signal strength. Lloyd-Hart et al[4] have shown that phase diversity techniques can yield good wavefront data from the sequence of out-of-focus images recorded as a fast movie in a fixed focal plane. Salinari[5] is exploring a geometric method that exploits the lateral image shift with height that arises for a bacon projected from alongside the telescope. The solution considered here is dynamic refocus of the telescope to follow a pulse rising from a centered projector.

Some adaptive optics systems already use a vibrating diaphragm mirror to induce curvature changes in the wavefront. This method is not suitable for our purpose, though, because the required amplitudes for the 8.4 m LBT apertures, $\geq 100\mu m$ in sagittal depth, are inconveniently large. The method I have in mind is to change the path length in a fast-converging beam, by reflection off a moving flat. The analysis below is cast in terms of such longitudinal motion of the beacon image.

For convenience, the next section gives the character of the projected Rayleigh pulse, and a table of the expected fluxes as a function of height. These results follow the treatment given by Angel and Lloyd-Hart.[1] In the third section the range gates and fluxes corresponding to a given blur circle are derived for fixed focus and refocus at constant speed.

2. DEPTH OF FIELD OF THE PROJECTED BEAM AND RAYLEIGH BEACON STRENGTH

Consider a diffraction limited laser projected through a beam expander, and focused to a point high in the atmosphere. Neglecting for the moment the effect of seeing, if a gaussian beam is brought to a focus with a waist of width $N\lambda$, the longitudinal depth of focus is $N^2\lambda$.[6] Let us suppose that a beacon is focused at height h_f and that the beam is projected such that the waist at this height subtends the seeing width, θ_s i.e. $N\lambda/h_f = \theta_s$. Remembering that $\theta_s = \lambda/r_0$ and solving for N, we find for the depth of focus Δh

$$\Delta h = N^2\lambda = \frac{{h_f}^2\lambda}{{r_0}^2}. \tag{1}$$

Because Fried's length r_0 varies as $\lambda^{6/5}$, the greatest depth of field is achieved at shortest wavelength. For a 360 nm Rayleigh beacon at $h_f = 25km$, suppose we require the beacon to subtend 0.6 arcsec, appropriate for good seeing, $(r_0(360nm) = 12.4cm)$. We find $\Delta h = 15\,km$. The sharp focus thus extends from 17 to 32 km. Such a beam requires a projected gaussian width of 8 cm, and the waist at 25 km is also 8 cm diameter. The rising pulse will subtend less than 1 arcsec from 17 km on up. Since the projected width is less than r_0, seeing will affect significantly only beam jitter, and short exposures will remain diffraction limited.

The geometry of the scattered beacon rays returning to the main telescope is shown in figure 1. We show rays from a short laser pulse as it rises through heights h_i, h_f and h_u. Phase errors originating in the turbulent layer at height h_t are offset and magnified by an amount that depends on beacon angle and height. The beacon shown is tilted, as required for full sampling of the higher altitude turbulence for a tomographic solution from multiple lasers.

The photon flux from Rayleigh scattering is given in table 1, calculated for a l Joule ultraviolet pulse (351 nm) projected straight up. The atmospheric pressure in mbar as a function of height from Allen[7] is given in the second column. The 3^{rd} and 4^{th} columns give the back-scattered flux in photons/m^2 from 1 km path length and from all scattering above the listed height, from the lidar equation.[8] The one-way atmoshpheric transmission is taken to be 50%, appropriate for 351 nm at the LBT site. For comparison, a cw laser precisely tuned for resonance scattering yields $\sim$ 800,000 photons/m^2/Joule from a mesospheric sodium layer of median column density.[9]

Table 1. Rayleigh scattered fluxes back at the telescope in photons/m^2 as a function of height, for a projected pulse of 1 Joule at 351 nm.

h(km)	P(mbar)	return/km	total return from above h
40	2.9	8,000	34,000
35	5.8	21,000	95,000
30	12	58,000	260,000
25	25	180,000	740,000
20	55	600,000	2,300,000

3. FOCUSING ON A RISING PULSE

Suppose for the moment that a detector is attached to a telescope of diameter D and focal length f, at the focal position conjugate to height h_0, and the return from that height is sensed at time t_0 after the pulse is projected, i.e. $t_0 = 2h_0/c$. The image will be out of focus at other times t, the change in the longitudinal focus $i(t)$ given to good approximation by

$$i(t) - i(t_0) = \frac{2f^2}{c}\left(\frac{1}{t} - \frac{1}{t_0}\right). \tag{2}$$

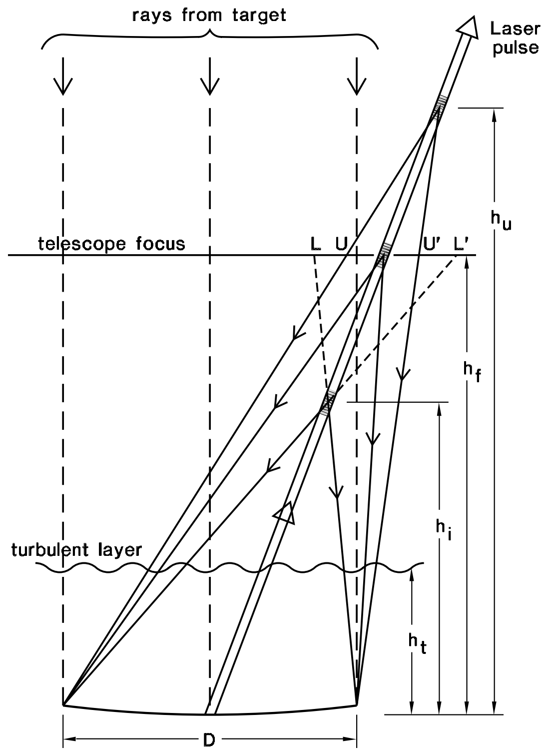

Figure 1.

Expanding in terms of the time difference $\Delta t = t - t_0$ we find the longitudinal shift in focus is given by

$$\Delta i = \frac{2f^2}{ct_0}\left(-\frac{\Delta t}{t_0} + \left(\frac{\Delta t}{t_0}\right)^2 - \left(\frac{\Delta t}{t_0}\right)^3 + ...\right). \tag{3}$$

Suppose the laser is projected from center of the telescope, as shown in figure 1. Focus error causes the image to expand into a circle centered about the optimum focus point. Its angular diameter, projected on the sky is given by

$$\theta_b = \left|\frac{D\Delta_i}{f^2}\right| \tag{4}$$

For conventional Rayleigh beacon operation with fixed focus, the linear term in Δ_i dominates, and we find $\Delta t = \pm\theta_b c t_0^2/2D$, corresponding to scattering over a full range of height

$$\Delta h_{(fixed\ focus)} = \frac{2{h_0}^2\theta_b}{D}. \tag{5}$$

Now suppose we introduce a mechanism by which the focal plane can be shifted dynamically. The longitudinal focus error could be removed completely if the shift were in accordance with eqn (3). Such motion could involve rather large accelerations. Let us consider the case in which the detector is moved with constant velocity to best match focus about time t_0. The longitudinal error now becomes

$$\Delta i(refocus) = \frac{2f^2}{ct_0}\left(-\frac{\Delta t}{t_0} + \left(\frac{\Delta t}{t_0}\right)^2 - \left(\frac{\Delta t}{t_0}\right)^3 + ...\right) - d_0 - v_d\Delta t, \tag{6}$$

where the offset d_0 and the detector velocity v_d are chosen to keep the blur circle diameter below the acceptable level for the longest time. v_d is the image speed at $t = t_0$, and d_0 is the focus offset at that time that results in a blur circle diameter θ_{max} i.e.

$$v_d = -\frac{f^2 c}{2{h_0}^2} \ \ and \ \ d_0 = \frac{f^2\theta_{max}}{D}. \tag{7}$$

With these values, the longitudinal focus error now becomes

$$\Delta i(refocus) = \frac{2f^2}{ct_0}\left(\left(\frac{\Delta t}{t_0}\right)^2 - \left(\frac{\Delta t}{t_0}\right)^3 + ...\right) - \frac{f^2\theta_{max}}{D}. \tag{8}$$

Provided $\Delta t/t_0 << 1$ we may neglect the terms higher than quadratic, and solve again for Δt corresponding to blur circle diameter θ_{max} i.e. $\Delta_i = +d_0$. We find in this case $\Delta_t = \pm\sqrt{ct_0{}^3\theta_{max}D}$, corresponding to scattering over a full range of height

$$\Delta h_{(refocus)} = 2h_0\sqrt{\frac{2h_0}{D}\theta_{max}}. \tag{9}$$

For the LBT with $D = 8.4m$, and taking a maximum blur circle diameter $\theta_{max} = 1$ arcsec, we obtain from equations (5) and (9) the height ranges given in table 2 below. For each range, we give the returned photon flux per Joule of projected pulse energy. We also give the constant speed v_d from equation 7 corresponding to an image formed at focal ratio f/1.

Stationary Focus				Constant speed refocus		
$t_0(\mu s)$	h_0(km)	Δh	phot/m^2/J	range (km)	phot/m^2/J	v_d(m/s)
100	15	0.25	500,000	2.7	8,000,000	47
133	20	0.44	270,000	6.1	3,600,000	26
167	25	0.7	126,000	8.5	1,900,000	17
200	30	1	60,000	11.2	840,000	12
233	35	1.4	28,000	14.1	350,000	9
267	40	1.8	14,000	17	120,000	7

These results are encouraging, in that they show an order of magnitude increase in flux is possible by dynamic refocus, with no degradation of image quality. The velocities, of order 10-20 m/sec, should be achievable without great difficulty. The height ranges for dynamic refocus are larger than the atmospheric scale height of about 7 km for values of $h_0 \geq 25\,km$. This means that a good fraction of all the scattered flux above the height at which the gate is opened is collected. For $h_0 = 25km$, for example, the gate for good focus is opened from 21 to 29 km, with a return flux of 1,900,000 photons/m^2/Joule. The entire return from above 21 km is not much more, as we see from table 1.

4. DISCUSSION

The sampling of high altitude turbulence required for tomography depends of the height of the laser beacons used and the field of view. For example, if a field of one arc minute is the goal, and there is a high layer of turbulence 8 km above the telescope, we need to measure it over a the 10.7 m diameter traversed by different rays to the 8.4 m aperture. Sodium beacons would each sample about 7.7 m diameter circles, while Rayleigh beacons 25 m above the telescope will sample 5.7 m circles. For the same degree of sampling, about twice as many Rayleigh beacons will be needed. This would seem to be a small price to pay for the convenience of using lasers that are commercially available, relatively simple, reliable and much less expensive.

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